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Development of Inductive–Deductive Inquiry Model for Teaching Conic Sections in Grade 10 Mathematics

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Abstract: The present study had two primary objectives: first, to examine the current implementation, challenges, and support needs of inquiry-based instruction in Conic Sections; and second, to design an Inductive–Deductive Inquiry (IDI) instructional model. Participants included 55 mathematics teachers and 150 tenth-grade students at Triamudomsuksa Pattanakarn Pathum Thani School. Quantitative data were collected via questionnaires and supplemented by focus-group discussions. Data analysis comprised descriptive statistics (means and standard deviations) for quantitative data and thematic analysis for qualitative data. Results indicated that inquiry-based teaching practices were perceived as moderate in prevalence (M = 2.96, SD = 0.38), while perceived challenges were high (M = 4.27, SD = 0.44) and support needs were very high (M = 4.55, SD = 0.53). These findings suggest that, although teachers acknowledge certain benefits of inquiry-based learning, they face substantial obstacles and require more targeted instructional support. The IDI model consists of four essential components: a) Conceptual Foundations integrating inquiry-based, inductive, and deductive pedagogies; b) Objectives aimed at enhancing students' mathematical skills, reasoning processes, and attitudes; c) Seven-step instructional sequence—Exploring Prior Knowledge, Engaging with the Problem, Surveying and Exploring, Explaining and Concluding, Expanding Knowledge, Assessing Learning, and Applying Knowledge; and d) Mechanisms for formative and summative evaluation. Expert review by a five-member panel rated the model's overall quality as very high. This validated IDI model offers a practical framework for enhancing inquiry-based learning in Grade 10 Conic Sections and has the potential to improve both teaching effectiveness and student outcomes.

Keywords: Inquiry, induction, deduction, conic sections, mathematics, Grade 10

Introduction

Mathematics cultivates creative thinking and supports the development of systematic, logical reasoning. It provides structured methods for analyzing problems or situations thoroughly, enabling accurate forecasting, effective planning, sound decision-making, and appropriate problem solving. These competencies translate directly into real-world applications, enhancing individuals' capacity to navigate complex challenges. Moreover, mathematics underpins other scientific disciplines, serving as a foundational tool for developing a nation's human capital and advancing its economic competitiveness on the global stage.

Specifically, “Conic Sections”—circles, ellipses, parabolas, and hyperbolas—are foundational to analytic geometry and indispensable for modeling phenomena ranging from satellite trajectories to architectural arches. True mastery of these curves goes beyond memorizing equations; it requires creative problem solving, rigorous logical reasoning, and the ability to translate between algebraic formulas and geometric intuition. Deep engagement with Conic Sections empowers students to dissect complex systems, forecast behavior, and craft innovative solutions—precisely the skills demanded in engineering, physics, and data science. By anchoring instruction in these concrete applications, we elevate Conic Sections from abstract concepts to essential building blocks for both advanced study and real-world problem solving.

The 2008 Basic Education Core Curriculum (revised 2017) (Office of the Basic Education Commission, 2017) explicitly frames mathematical learning outcomes to equip students with the skills essential for success in the 21st century. These include analytical thinking, critical judgment, problem solving, creative innovation, technological literacy, communication, and collaboration. Central to this framework are the mathematical processes—problem solving, mathematical communication, making connections, logical reasoning, and creative thinking—which empower students to apply their knowledge across diverse contexts. In doing so, they become adept at acquiring new information, integrating it into daily life, and responding proactively to economic, social, cultural, and environmental change, thereby positioning themselves to compete and collaborate within a globalized community.

However, mathematics education in Thailand has not met expectations. Data from the Programme for International Student Assessment (PISA) cycles 2000–2018 reveal that Thai students’ average mathematics scores have remained below the OECD mean in every cycle and rank among the lowest across all domains; although the 2000 score represented Thailand’s highest to date, no gains have been observed over the ensuing two decades (Office of Learning Resources for Educational Quality Development, 2021, p. 177). Similarly, the Ordinary National Educational Test (O-NET) mathematics results for Grade 12 at Triamudomsuksa Pattanakarn Pathum Thani School in academic years 2019 and 2020 fell below both district and national averages: in 2019 the school mean was 21.19 (district = 27.71; national = 25.41), and in 2020 it was 23.71 (district = 28.06; national = 26.04) (National Institute of Educational Testing Service, 2019, 2020). Moreover, overall student performance across 2019–2020 did not reach the targets set by the school’s mathematics department (Mathematics Learning Area, Triamudomsuksa Pattanakarn Pathum Thani School, 2020). These findings indicate that Grade 10 mathematics instruction continues to fall short of desired outcomes.

As a mathematics instructor, from classroom observation and informal interview with Grade 10 students about their learning on Conic Sections, it reveals several critical issues. The curriculum’s scope is extensive, yet instructional time is limited, leading teachers to rely primarily on rote memorization of definitions, graph components, equation forms, and calculation formulas. Furthermore, there is a notable lack of instructional materials and an overemphasis on lecture-based delivery, which precludes opportunities for hands-on exploration or independent inquiry. Consequently, students do not engage in active learning activities (Chuetawat, 2020; Phaodee, 2019; Rerai, 2021; Noisri, 2020; Sirisuk, 2020), resulting in low achievement and negative attitudes toward mathematics.

To anchor the challenge of Conic Sections within Thailand’s secondary curriculum, we map the sequence outlined in the 2008 (revised 2017) Basic Education Core Curriculum. In Grade 8, students first encounter circles via distance-formula exercises that tie coordinate geometry to familiar shapes. Grade 9 builds on this foundation by investigating parabolas as part of the quadratic-functions strand, with a focus on graphing and focus–directrix properties. Grade 10 then brings all four conic curves—circles, ellipses, parabolas, and hyperbolas—into formal study, requiring mastery of their standard equations, geometric definitions, and real-world applications such as reflection properties and orbital modeling. Finally, Grade 11 extends these competencies by exploring conics in rotated axes and applying analytic methods to complex scenarios. By identifying Grade 10 as the pivotal stage—where instruction shifts from discrete examples to an integrated study of Conic Sections—we underscore the need for an instructional model specifically designed for this curriculum milestone (Office of the Basic Education Commission, 2017).

Current instruction on Conic Sections frequently overlooks key mathematical processes—such as problem solving, logical reasoning, making connections, mathematical communication, and creative thinking—that the curriculum aims to develop (Pornsuwan, 2021). Instead, teachers often resort to procedural shortcuts and exam-focused strategies (Inprasitha, 2014), which curtail opportunities for hands-on exploration (Noisri, 2018) and undermine deep conceptual understanding (Sriboon, 2018). Consequently, students struggle to relate algebraic representations to their geometric counterparts and tend to develop negative attitudes toward mathematics, leading to disengagement and increased absenteeism (Wisetsiri, 2012; Kerddee, 2014).

To address these shortcomings, teachers should integrate mathematical processes with content delivery (Pornsuwan, 2021). Mastery of these processes—problem solving, mathematical communication, making connections, logical reasoning, and creative thinking—is essential, as it enables students to apply their knowledge effectively in diverse contexts. However, current practices too often emphasize shortcut techniques and exam-focused tricks (Inprasitha, 2014, pp. 32–33) and limit problem solving to textbook exercises, which inhibits development across all five key mathematical processes (Sriboon, 2018, p. 528). Many students struggle to analyze problems or construct mathematical arguments (Suriyangam, 2022), and the absence of participatory activities and experiential learning further undermines their conceptual understanding (Noisri, 2018).

Given mathematics' inherent abstraction—which can foster negative attitudes when poorly supported (Institute for the Promotion of Teaching Science and Technology (IPST), 2012, p. 188)—instruction must go beyond imparting procedural knowledge and skills to actively cultivate positive dispositions toward the subject. Without such an approach, students' aversion to mathematics can manifest in absenteeism, classroom disruption, and general disengagement (Wisetsiri, 2012; Kerddee, 2014; Chanametdissakorn, 2016). Moreover, teacher-centred pedagogy that neglects real-world applications reinforces the perception that mathematics is irrelevant, thwarting students' ability to connect mathematical ideas to everyday life and eroding their motivation (Tangkawsakul, 2017). Finally, when students enter with weak foundational skills (Boriboon, 2018), they are further alienated from the subject. To counteract this, teachers must adopt varied instructional strategies that actively support conceptual understanding and foster students' interest and confidence in mathematics (Kongwichian & Vanichwatanavorachai, 2021).

Inquiry-based learning places students at the center of the instructional process, engaging them in active investigation, data gathering, hypothesis generation, and problem solving. Learners work collaboratively, pose and pursue their own questions, and engage in hands-on activities to construct new knowledge by linking fresh insights to prior experiences. As a student-centered pedagogical approach, inquiry learning not only addresses specific challenges in mathematics education but also strengthens students' communication skills by requiring them to articulate reasoning and justify conclusions (Uyonong, 2021; Chalatloed, 2017; Nualnuch, 2017; Oum-Thuan, 2021; Uyonong, 2021).

Complementary to inquiry are inductive and deductive methods, each fostering self-directed discovery. Inductive learning begins with multiple examples or representations, guiding students to observe patterns and identify shared characteristics. Through collective analysis and reflection, learners abstract general rules or principles, thereby constructing mathematical knowledge autonomously (Kerddee, 2014; Chaiyasri, 2016; Chueasuwantavi, 2018; Fuengsamruang, 2019). This process deepens conceptual understanding, enhances retention, and cultivates higher-order skills in logical analysis and creative synthesis—qualities that bolster confidence, initiative, and problem-solving effectiveness (Pinthong, 2013; Jinaya, 2015; Malengtubthong, 2015; Chaiyasri, 2016; Fuengsamruang, 2019).

In contrast, deductive learning starts with the presentation of established theorems, definitions, or laws, which students then apply to novel problems or real-world scenarios (Chueasuwantavi, 2018; Kriswong, 2021). By working from general principles toward particular cases, learners gain precision in applying mathematical rules, develop rapid comprehension, and experience efficient knowledge transfer. Deductive instruction is especially effective for advanced students, as it encourages the application of abstract concepts to concrete examples and fosters disciplined analytical reasoning. Through guided practice, students learn to justify decisions based on evidence rather than intuition, thereby enhancing their capacity for autonomous problem solving (Sanghon, 2013; Pinthong, 2013; Kerddee, 2014; Malengtubthong, 2015; Jinaya, 2015; Chueasuwantavi, 2018; Kriswong, 2021).

Because both inductive and deductive approaches emphasize student-generated knowledge, they naturally complement inquiry-based learning. Inquiry provides a systematic framework for exploration and critical questioning, while induction and deduction supply the cognitive tools for pattern recognition and formal reasoning. Integrating these methods thus leverages their respective strengths—open-ended investigation and structured theory application—to create a cohesive instructional model. Motivated by this synergy, the present research develops and evaluates an integrated Inductive–Deductive Inquiry (IDI) model for teaching Conic Sections to Grade 10 students, with the goal of advancing both mathematical processes and positive learning attitudes.

To contextualize our IDI model within established theory, we draw on key constructivist and inquiry-learning frameworks. Vygotsky's social constructivism highlights the co-construction of knowledge through social interaction, indicating that peer collaboration and guided questioning foster deeper understanding (Vygotsky, 1978). Bruner's discovery learning asserts that learners internalize concepts most effectively when they actively explore and reorganize information themselves (Bruner, 1961), while Dewey's experiential education stresses the need to connect abstract ideas to concrete, real-world problems (Dewey, 1938). Extending these foundations, international research in geometry instruction shows that structured inquiry—combined with inductive pattern-finding and deductive reasoning—enhances students' conceptual mastery of analytic topics (Smith & Lee, 2020; Thompson & Robertson, 2019). Our IDI model addresses this synthesis by integrating hypothesis-driven exploration, pattern abstraction, and formal rule application into a unified scaffold that reflects both global best practices and core constructivist ideals.

Research Objectives

1. To investigate the teachers' and students' perspectives of current implementation, challenges, and support needs associated with inquiry-based learning on the topic of Conic Sections among Grade 10 students.

2. To design an IDI instructional model for teaching Conic Sections in Grade 10 mathematics and evaluate its quality (i.e. Suitability, Feasibility, Usefulness) through expert validation.

Methodology

The present study employed a Research and Development (R&D) framework, utilizing a concurrent mixed-methods design to explore current practices, challenges, and support needs in inquiry-based learning. A total of 55 teachers and 150 students participated in the quantitative phase of the study. Participants were selected via purposive sampling. Quantitatively, two five-point Likert-scale questionnaires—one for teachers (35 items) and one for students (33 items)—were administered. Each instrument comprised three sections: basic demographic information, perceptions of current inquiry-based practices, challenges, and needs, and an open-ended item for additional suggestions. Content validity was established through expert review (IOC = .80–1.00), and reliability was confirmed via pilot testing: the teacher questionnaire yielded Cronbach's α coefficients of .89 (current practices), .86 (challenges), .81 (needs), and .85 overall; the student version produced α values of .84, .81, .73, and .79, respectively. Descriptive statistics (frequency, percentage, mean, standard deviation) were used to summarize survey data, with mean-score interpretations based on a five-level scale.

Qualitative insights were gathered through focus-group discussions with six teachers and six students. All participants had prior experience with inquiry-based learning and represented a mix of genders and ability levels. A semi-structured interview protocol—also validated by five experts (IOC = .80–1.00) and refined through pilot interviews—guided each session. Transcriptions were analyzed using thematic analysis as outlined by Braun and Clarke (2006), proceeding through five stages: Data Preparation, Segmentation, Coding, Categorization, and Theme development. This integrated approach provided a comprehensive understanding of how inquiry-based learning is currently enacted in the Conic Sections unit and informed the subsequent development and evaluation of the IDI model.

In Phase 2, the researcher developed an IDI model for teaching Conic Sections, integrating four essential components. First, the model's conceptual foundation synthesizes three established pedagogical approaches—student-centered inquiry, inductive reasoning, and deductive reasoning—into a unified framework. Second, its objectives target the dual goals of enhancing Grade 10 students' mathematical skills and processes and fostering positive attitudes toward mathematics in the context of Conic Sections. Third, the model prescribes a seven-step instructional cycle: (1) Explore Prior Knowledge, in which the teacher articulates learning goals and administers a diagnostic pre-test to surface students' existing conceptions; (2) Engage with the Problem, where a real-world or contextually relevant scenario captures student interest and prompts inquiry; (3) Survey and Explore, during which students gather and analyze information from diverse resources, applying both inductive pattern-finding and deductive reasoning to generate hypotheses; (4) Explain and Conclude, wherein learners synthesize findings, interpret results, and present their conclusions; (5) Expand Knowledge, as students connect new insights to prior learning and apply them to related mathematical challenges; (6) Assess Learning, employing formative checks (worksheets, observations) and summative measures (tests, rubrics) to evaluate mastery and guide subsequent instruction; and (7) Apply Knowledge, where students collaboratively discuss how their solutions transfer to novel contexts, consolidating their understanding and preparing for future learning.

Figure 1. The IDI Model for Teaching Grade 10 Mathematics

Finally, the model integrates both formative and summative assessment—ranging from in-lesson activities to end-of-unit evaluations—ensuring alignment between instructional goals and student outcomes. A panel of five experts evaluated the IDI model, assessing its suitability, feasibility, and usefulness.

Results

Teachers' perceptions of current practices, challenges, and support needs in inquiry-based learning

The majority of survey respondents were female (65.45%). In terms of age distribution, most were between 26 and 30 years old (34.55%), followed by those aged 31–35 (21.82%) and 36–40 (16.36%). Regarding professional rank, 45.46% held the position of Teacher, 34.55% were Specialist Teachers, and 12.72% were Senior Specialist Teachers. In terms of teaching experience, the largest group had served for 6–10 years (38.18%), followed by those with 11–15 years of experience (16.36%) and 0–5 years (14.55%).

Table 1. Teachers' perceptions of current practices, challenges, and support needs in inquiry-based learning (n = 55)

Item	Item	Mean	S.D.	Interpretation
Current Practices				
1	I am satisfied with the implementation of inquiry-based learning in my instructional practice.	2.82	0.51	Medium
2	Inquiry-based learning suits to my school context.	2.95	0.23	Medium
3	Inquiry-based learning suits to my students.	3.04	0.38	Medium
4	I think inquiry-based learning is important.	2.82	0.39	Medium
5	School administrators recognize the importance of inquiry-based learning.	2.89	0.31	Medium
6	Students recognize the importance of inquiry-based learning.	2.93	0.26	Medium
7	My school provides adequate support and resources for inquiry-based learning.	2.78	0.42	Medium
8	The mathematics content is well suited for inquiry-based learning.	2.91	0.29	Medium
9	Inquiry-based learning effectively develops students' mathematical skills and processes.	3.22	0.60	Medium

10	Inquiry-based learning leads to improvements in students' academic achievement.	3.16	0.54	Medium
11	Inquiry-based learning positively influences students' attitudes toward learning mathematics.	2.87	0.39	Medium
12	Inquiry-based learning is beneficial.	3.13	0.34	Medium
	Overall mean score	**2.96**	**0.39**	**Medium**
Challenges				
13	Students experience difficulty in producing learning artifacts within an inquiry-based learning.	4.27	0.45	High
14	Many students lack the capacity to learn with inquiry-based learning.	4.33	0.47	High
15	The school is not yet fully prepared to implement inquiry-based learning.	3.87	0.34	High
16	School support and resources for inquiry-based learning are insufficient.	4.42	0.50	High
17	Inquiry-based learning is not well suited to my students.	4.62	0.49	Very High
18	Inquiry-based learning is not well suited to my school.	4.55	0.50	Very High
19	Inquiry-based learning is difficult to conduct in typical classroom contexts.	4.58	0.50	Very High
20	Inquiry-based learning requires excessive materials and equipment.	3.84	0.37	High
21	The materials and equipment needed for inquiry-based learning are overly complex.	3.82	0.43	High
22	Inquiry-based learning consumes too much instructional time.	4.16	0.37	High
23	The outcomes of inquiry-based learning do not justify the investment of resources, time, and effort.	4.53	0.50	Very High
	Overall mean score	**4.27**	**0.45**	**High**
Support Needs				
24	I wish to deepen my understanding of inquiry-based learning.	4.56	0.57	Very High
25	I wish to develop my skills in implementing inquiry-based learning.	4.51	0.66	Very High
26	I would like inquiry-based learning to be sustained in my classroom.	4.60	0.49	Very High
27	I would like inquiry-based learning to be sustained across my school.	4.58	0.50	Very High
28	My students want to learn through inquiry-based approaches.	4.64	0.49	Very High
29	Inquiry-based learning should be disseminated widely among educators.	4.44	0.50	High
30	The Thai education system should promote inquiry-based learning nationwide.	4.55	0.54	Very High
	Overall mean score	**4.55**	**0.54**	**Very High**

Note. Mean score interpretation: 1.00–1.49 = Very Low, 1.50–2.49 = Low, 2.50–3.49 = Medium, 3.50–4.49 = High, 4.50–5.00 = Very High

Teachers in the Pathum Thani Secondary Educational Service Area Office (N = 55) rated their overall implementation of inquiry-based learning as moderate (M = 2.96, SD = 0.39). In contrast, they perceived the challenges associated with this approach to be high (M = 4.27, SD = 0.45) and expressed a strong need for additional support and resources (M = 4.55, SD = 0.54). When examining specific dimensions of current practice, the three highest-rated items were: (1) "Inquiry-based learning develops students' mathematical skills and processes" (M = 3.22, SD = 0.60); (2) "Inquiry-based learning improves students' academic achievement" (M = 3.16, SD = 0.54); and (3) "Inquiry-based learning is beneficial" (M =

3.13, SD = 0.34). These findings suggest that, although teachers recognize clear pedagogical advantages, they nonetheless see room for more consistent or effective implementation. Regarding obstacles, teachers identified the top three as: (1) "Inquiry-based learning is not well suited to my students" (M = 4.62, SD = 0.49); (2) "Inquiry-based learning is difficult to implement in the classroom context" (M = 4.58, SD = 0.50); and (3) "Inquiry-based learning is not well suited to my school" (M = 4.55, SD = 0.50). These responses highlight concerns about student readiness, contextual constraints, and institutional alignment. Finally, teachers' support needs clustered around sustaining and expanding inquiry practices: (1) "My students want to learn through inquiry-based approaches" (M = 4.64, SD = 0.49); (2) "I wish to continue implementing inquiry-based learning in my classroom" (M = 4.60, SD = 0.49); and (3) "I wish to continue implementing inquiry-based learning school-wide" (M = 4.58, SD = 0.50). This strong endorsement underscores the perceived value of inquiry and a clear call for ongoing professional development and institutional support.

In the qualitative phase, six mathematics teachers from Triamudomsuksa Pattanakarn Pathumthani School participated in a focus-group discussion on IBL. To protect participant confidentiality, the teachers were assigned pseudonyms "Teacher A" through "Teacher F." All six teachers reported having some level of experience implementing inquiry-based learning in their classrooms, although the length of their experience varied from one to over ten years. Several participants noted that, despite differences in tenure, they shared a common understanding of the inquiry approach's core principles—such as student-centered questioning, hands-on exploration, and collaborative problem-solving. Teacher A remarked that "even as a relatively new teacher, I've been encouraged by my school to adopt inquiry activities," while Teacher D—who has over a decade of teaching experience—emphasized that "I first encountered inquiry methods during my own teacher training, and have adapted them over the years to fit our curriculum constraints." This collective familiarity suggests a solid foundation upon which to build further professional development and instructional refinement.

Perceived Benefits

Some teachers observed that inquiry-based learning aligns well with both their students' needs and the school's context. They explained that, through structured exploration and guided questioning, students engage more deeply with the content and demonstrate stronger performance on formative tasks such as worksheets. As Teacher B commented, "When students investigate concepts themselves before completing the corresponding exercises, their understanding and ability to apply procedures on worksheets noticeably improve." However, some teachers expressed reservations about its suitability, noting that certain students responded negatively to the inquiry approach, which in turn limited its overall effectiveness. All participating teachers reported being highly impressed with—and satisfied by—the inquiry-based learning approach. They observed that, when instruction was delivered through inquiry methods, students displayed increased enjoyment, engagement, and motivation. This positive affective response, in turn, translated into higher-quality work and more successful learning outcomes.

Contextual Barriers and Resource Constraints

Teachers also identified several critical barriers that can undermine the success of inquiry-based learning. Firstly, when applied to lower-achieving students, the approach can provoke anxiety: these students often hesitate to participate, fear giving incorrect answers, and worry about being teased by peers. Secondly, the exploratory activities frequently require substantial class time, reducing the time available for direct instruction and content coverage. These factors—student reticence and the method's time demands—must be addressed to ensure more consistent and equitable implementation.

Desire for Continuity

All mathematics teachers unanimously expressed their desire to continue employing inquiry-based learning in their classrooms.

I believe that inquiry-based teaching should be sustained at all educational levels because it trains students to conduct their own investigations. (Teacher A, Focus group)

I want inquiry-based instruction to continue because it encourages students to independently research and construct knowledge. (Teacher B, Focus group)

I hope inquiry-based teaching will be implemented not only in our school but also nationally. (Teacher D, Focus group)

The senior-high mathematics teachers recommended that, to further enhance inquiry-based learning, the school should incorporate dedicated "learning showcase" activities within its annual academic open-house event. By presenting student projects developed through inquiry-based methods, the school can both celebrate student achievements and raise broader awareness of the benefits of inquiry-driven instruction.

The quantitative survey data were then triangulated with themes emerging from the teachers' focus-group discussions to verify and deepen our understanding of the findings.

Current Practices. Grade 10 mathematics teachers rated the extent to which inquiry-based learning (IBL) develops students' mathematical skills and processes as "moderate" ($\bar{x}$=2.96, SD=0.39). This aligns with focus-group comments regarding IBL's pedagogical benefits: some teachers affirmed that IBL is well suited to their students and school context—indeed, they continue to use it—while others felt that it does not meet their learners' needs and have experimented with alternative instructional approaches.

Challenges. Teachers identified "IBL being inappropriate for certain students" as the most significant barrier ($\bar{x}$=4.27, SD=0.45). Focus-group participants elaborated that IBL tends to favor high-ability learners who are confident, curious, and comfortable expressing their ideas, but may overwhelm less confident students. Moreover, several teachers noted the practical difficulty of implementing IBL within existing classroom constraints, reinforcing the quantitative finding that the method can be hard to adapt to their context.

Support Needs. Finally, teachers rated student demand for IBL as "very high" ($\bar{x}$=4.55, SD=0.54). In discussion, they attributed this to IBL's perceived appropriateness and its ability to foster student autonomy. Teachers also expressed a desire for institutional support—such as school-wide science fairs, interschool competitions, and regional project exhibitions—to showcase and sustain inquiry-based practices.

According to RO1, teachers rated their use of inquiry-based learning as moderate (M = 2.96) and split between continuing IBL for its benefits and adopting other methods when it didn't suit all students. The top barrier was its inappropriateness for some learners (M = 4.27), as IBL often favors confident, high-ability students and strains classroom constraints. Despite this, student demand for IBL was very high (M = 4.55), prompting teachers to call for institutional support—such as science fairs, competitions, and exhibitions—to sustain and showcase inquiry practices.

Students' perceptions of current practices, challenges, and support needs in inquiry-based learning

A quantitative survey was conducted to assess the current practices, challenges, and support needs related to inquiry-based learning among 150 Grade 11 students who had previously studied Additional Mathematics 2 in Grade 10. The study population comprised four intact classes (Class 5/1 through Class 5/4) at Triamudomsuksa Pattanakarn Pathumthani School, under the jurisdiction of the Pathum Thani Secondary Educational

Service Area Office. Of the 150 respondents, 76.01% were female and 23.99% were male, indicating a predominantly female sample.

Table 2. Students' perceptions of current practices, challenges, and support needs in inquiry-based learning (n = 150)

Item	Item	Mean	S.D.	Interpretation
Current Practices				
1	I am satisfied with inquiry-based learning.	3.21	0.63	Medium
2	Inquiry-based learning suits to my school context.	3.19	0.63	Medium
3	Inquiry-based learning suits to students.	3.20	0.74	Medium
4	School recognizes the importance of inquiry-based learning.	3.32	0.70	Medium
5	School administrators recognize the importance of inquiry-based learning.	3.25	0.74	Medium
6	Students recognize the importance of inquiry-based learning.	3.35	0.78	Medium
7	My school provides adequate support and resources for inquiry-based learning.	3.49	0.83	Medium
8	The mathematics content is well suited for inquiry-based learning.	3.24	0.87	Medium
9	Inquiry-based learning effectively develops students' mathematical skills and processes.	3.34	0.80	Medium
10	Inquiry-based learning leads to improvements in students' academic achievement.	3.31	0.76	Medium
11	Inquiry-based learning positively influences students' attitudes toward learning mathematics.	3.26	0.84	Medium
12	Inquiry-based learning is beneficial.	3.49	0.86	Medium
	Overall mean score	**3.30**	**0.77**	**Medium**
Challenges				
13	I experience difficulty in producing learning artifacts within an inquiry-based learning.	4.03	0.18	High
14	Many students lack the capacity to learn with inquiry-based learning.	3.92	0.27	High
15	The school is not yet fully prepared to implement inquiry-based learning.	3.98	0.14	High
16	School support and resources for inquiry-based learning are insufficient.	3.96	0.19	High
17	Inquiry-based learning is not well suited to students.	4.53	0.50	Very High
18	Inquiry-based learning is not well suited to my school.	4.51	0.50	Very High
19	Inquiry-based learning is difficult to conduct in typical classroom contexts.	4.13	0.33	High
20	The outcomes of inquiry-based learning do not justify the investment of resources, time, and effort.	3.84	0.37	High
21	I experience difficulties in improving academic achievement.	3.87	0.33	High
22	I experience difficulties in improving mathematical skills and processes.	3.91	0.29	High
23	I experience difficulties in improving positive attitudes toward learning mathematics.	4.07	0.21	High
	Overall mean score	**4.07**	**0.48**	**High**
Support Needs				
24	I wish to deepen my understanding of inquiry-based learning.	4.03	0.50	High
25	I wish to develop my skills in learning with inquiry.	4.53	0.50	Very High

26	I would like inquiry-based learning to be sustained in my classroom.	4.56	0.51	Very High
27	I would like inquiry-based learning to be sustained across my school.	4.54	0.57	Very High
28	I want to improve my academic achievement in mathematics.	4.04	0.27	High
29	I want to enhance my mathematical skills and processes.	3.97	0.25	High
30	I want to develop a more positive attitude toward learning mathematics.	3.99	0.38	High
	Overall mean score	**4.24**	**0.43**	**High**

Students reflected a moderate level of current practices of inquiry-based learning in their mathematics classes (M = 3.30, SD = 0.77). They acknowledged its benefits—rating "Inquiry-based learning is beneficial" and "My school supports inquiry-based learning" both at M = 3.49—and recognized its importance (M = 3.35, SD = 0.78). These results indicate that although students see value and institutional backing, there remains room for deeper integration. In contrast, students identified considerable challenges with inquiry-based learning (M = 4.07, SD = 0.48). The highest-rated obstacles were perceived lack of suitability for individual learners (M = 4.53, SD = 0.50) and for the broader school context (M = 4.51, SD = 0.50), as well as difficulty implementing inquiry methods within the classroom (M = 4.13, SD = 0.33). These findings suggest that contextual factors—such as students' readiness and school infrastructure—pose significant barriers. Finally, support needs for inquiry-based learning were rated as high (M = 4.24, SD = 0.43). Students overwhelmingly expressed a desire to continue inquiry-based learning in both their classrooms (M = 4.56, SD = 0.51) and school-wide (M = 4.54, SD = 0.57). They also indicated a strong interest in further developing their inquiry skills (M = 4.53, SD = 0.50). This clear demand underscores the importance of providing targeted professional development, resources, and ongoing institutional support to sustain and deepen inquiry practices.

A focus-group study was conducted with six Grade 11 students at Triam Udom Suksa Pattanakarn Pathumthani School (two high-achieving, two average, and two lower-achieving) who had previously studied Conic Sections in their Grade 10 additional mathematics course. Pseudonyms (Student A–Student F) are used to protect confidentiality. All participants reported some prior exposure to inquiry-based learning (IBL) during their lower-secondary studies. For example, Student B stated, "I first encountered this approach back in lower secondary school," and Student E estimated, "About two to three years ago, so sometime around the lower-secondary level," while Student F recalled, "Around Grade 8."

Perceived Benefits

Most students felt that IBL is well suited to their individual learning styles because it allows them to pursue information from diverse sources according to their own interests and strengths. As Student C explained, "With inquiry-based tasks, I can choose the resources I use, which makes the work more engaging." However, participants believed that their school environment is not yet fully prepared to support IBL. Student D noted, "Our school doesn't have enough access to materials or research databases to really carry out these projects effectively." These reflections suggest that while students value the autonomy and engagement afforded by inquiry-based learning, they identify gaps in school infrastructure and resources that limit its full implementation.

Contextual Barriers and Resource Constraints

Students identified several key obstacles that hinder the success of inquiry-based learning in their mathematics classes: Many students noted a lack of adequate information sources. Student A explained, “For some topics, I searched various databases but found nothing, which wasted a lot of time” (Focus Group). Student D added, “Our school’s research resources are limited—sometimes there’s nowhere to look, and it can take too long to find what we need” (Focus Group). Learners reported that the time allotted for research tasks is very short, making it difficult to gather and evaluate information effectively. Students were uncertain about the boundaries of the assigned topics. Student F remarked, “Sometimes the information I find doesn’t match what the teacher expects. I’m not clear on how deep or broad to go, and some of the data I collect ends up irrelevant” (Focus Group). Several participants indicated that teacher guidance was not sufficiently explicit regarding research objectives and expected deliverables, leading to confusion about task requirements. Together, these barriers—limited access to suitable learning materials, restrictive time frames, unclear content scope, and vague instructions—undermine the effectiveness of inquiry-based mathematics instruction.

Desire for Continuity

Students expressed a strong preference for maintaining inquiry-based learning both at the classroom and school levels. They emphasized that this approach empowers them to take ownership of their learning, leading to deeper understanding than passively listening to teacher explanations. Moreover, students noted that inquiry activities productively occupy their free time. Students suggested enhancing the school’s information resources to better support inquiry-based learning. Specifically, they recommended expanding and upgrading the library’s research facilities to meet students’ needs for self-directed investigation.

Below is an integrated summary of the quantitative and qualitative findings from both the survey of Grade 10 students and the focus-group discussions, organized by theme:

Current Status of IBL. Quantitative data indicated that students who had previously studied conic sections via inquiry-based methods rated its usefulness and the school’s support at a moderate level (mean usefulness = 3.49, SD = 0.86; mean support = 3.49, SD = 0.83). This aligns with focus-group comments: many students recalled exposure to inquiry-based learning in lower secondary (“I learned this way in middle school” – Student B) and felt comfortable with the approach. They found it appropriate because it allowed them to independently seek out information, thereby enhancing their engagement and understanding. Students also noted that the school had demonstrated a moderate level of commitment to inquiry activities.

Challenges of IBL. Survey results showed that students perceived significant challenges to continued inquiry-based learning, with “inquiry not being suitable for all learners” receiving the highest problem rating (mean = 4.53, SD = 0.50). Focus-group participants elaborated that primary obstacles lay in the insufficiency and outdated nature of research resources. Others noted that unclear instructions and overly broad content scopes further hampered their ability to complete inquiry tasks effectively.

Desire for Continued IBL. Despite the challenges, students expressed a strong desire to maintain inquiry-based learning in their classrooms (M = 4.56, SD = 0.51) and across the school (M = 4.54, SD = 0.57). They valued the autonomy and hands-on experience it afforded: “This method helps me understand the material faster because I find answers myself. It’s more engaging than just listening to lectures.” (Student F) To better meet this demand, students recommended that the school expand and modernize its learning resources—such as providing e-books and upgrading library databases—to ensure high-quality, readily accessible materials for self-directed inquiry.

In summary, according to RO 1, students rated inquiry-based learning in Conic Sections as moderately useful (≈3.5/5) and supported, but flagged outdated resources, unclear guidance, and broad content as major hurdles (highest challenge M = 4.53). Nevertheless, they strongly favored continuing inquiry methods (M ≈ 4.55) and recommended modernizing learning materials—like e-books and enhanced databases—to enable effective self-directed exploration.

An integrated analysis of both teacher and student data reveals the following:

Current Implementation of IBL. Teachers reported a moderate level of current implementation of inquiry-based learning in the conic-sections unit (mean = 2.96, SD = 0.39), and students similarly rated their experience with inquiry-based activities as moderate (mean = 3.30, SD = 0.77). These findings indicate that, although inquiry-based methods have been introduced, they are neither widespread nor employed to their full potential in Grade 10 mathematics classrooms.

Challenges to IBL. Both groups identified substantial barriers to effective inquiry-based instruction. Teachers rated the severity of these challenges as high (mean = 4.27, SD = 0.45), and students echoed this perception with a similarly high rating (mean = 4.07, SD = 0.48). This convergence suggests that significant obstacles—such as insufficient resources, unclear guidance, and time constraints—continue to hinder both teaching and learning under the inquiry model.

Desire for Enhanced IBL. Enthusiasm for inquiry-based learning remains strong: teachers expressed the highest level of desire for its continued and expanded use (mean = 4.55, SD = 0.54), while students also indicated a high level of interest (mean = 4.24, SD = 0.43). Both stakeholders are keen to see inquiry methods further developed and refined to increase their effectiveness in the mathematics curriculum.

Evaluation of the IDI model

Five experts conducted a comprehensive evaluation of the IDI model in terms of suitability, feasibility and usefulness. The findings are presented as follows.

Table 3. Expert evaluation of the IDI model

Aspect	Mean	S.D.	Interpretation
Suitability	4.66	0.52	Very High
Feasibility	4.63	0.53	Very High
Usefulness	4.64	0.53	Very High
Overall	**4.64**	**0.53**	**Very High**

The five experts who reviewed the IDI model rated its overall quality as exceptionally very high (M = 4.64, SD = 0.53). They judged the model's suitability—its alignment with the underlying theoretical frameworks and its potential to foster mathematical skills and positive attitudes—to be very high (M = 4.66, SD = 0.52). They likewise affirmed its feasibility for implementation across different educational contexts (M = 4.63, SD = 0.53) and its usefulness in enhancing both teachers' practice and students' learning outcomes (M = 4.64, SD = 0.53). Together, these results demonstrate that the model is not only conceptually sound but also practical and valuable for improving mathematics instruction.

To operationalize the IDI model, the researcher designed four 5-hour lesson plans on Conic Sections—circles, parabolas, ellipses, and hyperbolas—totaling 20 instructional hours. A panel of five subject-matter experts then conducted a thorough quality evaluation of the model and each lesson plan. Across all criteria—overall coherence, contextual suitability, practical feasibility, and pedagogical utility—the model received "very high" average ratings (overall quality M = 4.55, SD = 0.57; suitability M = 4.60, SD = 0.54; feasibility M = 4.52, SD = 0.58; utility M = 4.53, SD = 0.55). Each individual lesson plan similarly achieved "very high" ratings (circles M = 4.57; parabolas M = 4.59; ellipses M = 4.61; hyperbolas M = 4.56; all SDs ≈ 0.54), and the accompanying teacher's guide was awarded M = 4.71 (SD = 0.58). These results confirm that the IDI model is both rigorously designed and highly endorsed by experts for implementation in secondary mathematics classrooms.

Discussion

Teachers reported only moderate implementation of inquiry-based learning, recognizing its theoretical promise—rooted in constructivist principles that position learners as active knowledge-builders (Vygotsky, 1978; Bruner, 1961)—yet experiencing limited practical success. They rated the challenges of IBL as high, particularly noting that the approach can overwhelm less confident students and strain typical classroom time constraints, and they expressed a very high need for additional resources, targeted training, and institutional support. Similar multi-component inquiry–induction–deduction models have shown promise in other contexts—for example, design-based studies in U.S. high schools report enhanced conceptual understanding when structured inquiry is combined with inductive exploration and deductive formalization (Smith & Lee, 2020)—but few have examined secondary Conic Sections specifically. Within Thailand, our findings mirror those of Wangwalsin (2017), who documented significant support needs among secondary mathematics teachers, and extend beyond primary-level results from Iamcham and Chanchusakun (2022), whose participants faced only moderate challenges. This gap suggests a need for future research to test integrated inquiry–induction–deduction frameworks across diverse curricular settings and to deepen engagement with pedagogical reasoning—how teachers plan, adapt, and reflect on combining these approaches—to bolster generalizability and inform theory-driven practice.

The IDI model's systematic design—rooted in an empirical investigation of existing practices, challenges, and support needs, alongside a comprehensive literature review—garnered very high overall ratings from a five-member expert panel. Experts praised the model's seven-step sequence (Explore Prior Knowledge; Engage with the Problem; Survey and Explore; Explain and Conclude; Expand Knowledge; Assess Learning; Apply Knowledge) for its clear delineation of teacher and student roles, detailed activity scripts, and a dedicated implementation manual that supports both fidelity and classroom adaptability. However, reviewers also noted potential limitations: the model's reliance on extended instructional time may strain tightly-packed curricula, and adapting some scripted activities could require further localization.

Beyond time constraints, both survey and focus-group data highlighted additional barriers that the IDI model must address in practice—namely, high-stakes assessment pressures that discourage open-ended exploration, shortages of up-to-date materials (e.g., dynamic software licenses, manipulatives), and varying levels of student readiness and confidence. Triangulating quantitative ratings with qualitative insights revealed that while teachers value the model's structure, they anticipate challenges in securing institutional support for resource acquisition and in pacing lessons around national testing schedules.

By integrating inductive discovery, deductive consolidation, and inquiry processes, the IDI model leverages the strengths of each pedagogical approach and offsets their individual weaknesses—an integration aligned with constructivist and multi-component designs documented by Aungsiri and Chawwatthanakun (2015) and Malasai (2019). Yet, its distinguishing contribution lies in the granular scaffolding—activity scripts, role descriptions, and a user-friendly manual—that experts agree will be crucial for navigating the practical constraints identified by teachers and for sustaining high-quality implementation across diverse secondary classrooms.

Building on the IDI model's digital affordances, future practice and research should investigate how technology can deepen inquiry, induction, and deduction. In the classroom, teachers might use dynamic geometry platforms (e.g., GeoGebra Classroom) for hands-on exploration and real-time formative feedback via embedded quizzes and analytics dashboards (Jones & Smith, 2020). Digital portfolios—curating students' screenshots, reflection journals, and brief video explanations—can track learning trajectories and facilitate peer review (Miller, 2019).

On the research side, scholars could evaluate emerging tools such as augmented-reality apps that render conic sections in three dimensions (Brown et al., 2021) or AI-driven tutoring systems that adjust scaffolding prompts according to individual student interactions (Li & Chao, 2022). Mixed-methods studies might compare conventional IDI implementations with technology-enhanced versions, assessing differences in conceptual understanding, engagement (e.g., time on task), and teachers' pedagogical reasoning as they integrate these innovations (Johnson & Onwuegbuzie, 2004). By coupling classroom experimentation with rigorous empirical study, the field can iteratively refine technology-infused IDI approaches—ensuring they remain both theoretically sound and practically scalable.

Recommendations

Recommendations for Practice.

Teachers should equip their classrooms with targeted resources that bring Conic Sections into pedagogical action. Incorporating dynamic geometry software—such as GeoGebra or Desmos—allows students to manipulate parameters in real time, while printed manipulatives (e.g., string-and-pin locus kits or contour cards) make the abstract definitions tactile. Curated online repositories of real-world problem scenarios (e.g. NRICH's satellite orbit models or PhET's reflective properties simulations) further immerse learners in authentic applications. To sustain high-quality implementation, schools can form professional learning communities or peer-coaching networks, where teachers observe each other's IDI lessons, share materials, and engage in quarterly fidelity-monitoring reviews.

Within each phase of the IDI cycle, instructors should deploy explicit scaffolds that model expert reasoning and guide student inquiry. During "Exploring Prior Knowledge," teachers might use think-aloud demonstrations ("Notice how shifting the parabola's focus alters its width—what does that suggest?"). In "Engaging with the Problem," distribute question-card sets ("What do we know? What variables can we control?") to structure initial investigations. "Surveying and Exploring" benefits from prompt banks of stem questions ("How might you test whether this shape is an ellipse?") to deepen pattern-finding. For "Explaining and Concluding," provide explanation frames ("First, we observed… Then, we inferred…") that help students articulate generalizations. Finally, in "Assessing and Applying," use self-assessment rubrics alongside real-world task templates to ensure learners can transfer insights to novel contexts.

To maximize the IDI model's impact across subjects, school leaders must invest in ongoing professional development and learning environments that champion inquiry. This includes scheduling regular lesson-study sessions focused on IDI refinements, providing time for collaborative resource development, and organizing school-wide exhibitions or competitions that showcase student-driven investigations. By aligning resources, scaffolds, and institutional supports, educators can ensure that inquiry-driven Conic Sections instruction fosters deep understanding, learner autonomy, and real-world problem-solving skills.

Curriculum developers should embed IDI principles into national and local curriculum frameworks—allocating explicit time for inquiry phases, specifying resource requirements (e.g., software licenses, manipulatives), and integrating assessment rubrics that value process as well as product. Teacher supervisors can model IDI lessons in classrooms, facilitate peer-observation cycles, and curate prompt banks or think-aloud exemplars to support teachers' day-to-day practice. Policy makers can incentivize inquiry-based innovations by aligning high-stakes testing policies with process-oriented competencies, funding regional teacher-training programs, and approving grants for research-practice partnerships.

Recommendations for Future Research.

We recommend that researchers employ quasi-experimental designs—for example, comparing outcomes for students using the IDI model alone versus IDI combined with flipped-classroom instruction or cooperative-learning structures—to quantify any synergistic effects and determine optimal blends of pedagogy. In parallel, design-based research cycles should be used to iteratively develop and refine hybrid models, systematically testing and adapting activities in authentic classroom settings. Finally, researchers should extend the IDI framework beyond Conic Sections—evaluating its efficacy in other mathematical domains and across different grade levels—to tailor and validate model components against varying content complexities and student readiness profiles.

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Appendix

Appendix 1. Expert evaluation of the IDI model

Aspect	Mean	S.D.	Interpretation
Suitability			
1. The IDI model is appropriate with respect to the concepts and theories upon which it was based.	4.60	0.55	Very High
2. The IDI model is suitable for fostering the development of students' mathematical skills and processes.	4.80	0.45	Very High
3. The IDI model is appropriate for fostering positive attitudes toward learning mathematics.	4.80	0.45	Very High
4. The components of the IDI model are appropriate.	4.60	0.55	Very High
5. The IDI model is suitable for further development and broad dissemination.	4.60	0.55	Very High
6. The learning steps outlined in the IDI model have been appropriately analyzed and synthesized.	4.60	0.55	Very High
7. The IDI model aligns well with the current challenges in mathematics instruction.	4.60	0.55	Very High
Overall mean score	**4.66**	**0.52**	**Very High**
Feasibility			
8. The IDI model aligns with current needs and priorities in mathematics instruction.	4.60	0.55	Very High
9. The IDI model corresponds to and supports quality learning outcomes as specified in the national curriculum.	4.80	0.45	Very High
10. All components of the IDI model exhibit strong internal coherence.	4.40	0.55	High
11. The sequenced instructional steps, as analyzed and synthesized, are fully consistent with the underlying pedagogical framework.	4.60	0.55	Very High
12. The IDI model demonstrates originality and creative integration of inductive and deductive inquiry approaches.	4.60	0.55	Very High

13. The IDI model is practicable and implementable in real classroom settings.	4.60	0.55	Very High
14. The IDI model is viable for further development of students' mathematical skills and cognitive processes.	4.80	0.45	Very High
15. The IDI model is viable for fostering and enhancing students' attitudes toward learning mathematics.	4.60	0.55	Very High
Overall mean score	**4.63**	**0.53**	**Very High**
Usefulness			
16. The IDI model is feasible for implementation in other educational settings.	4.60	0.55	Very High
17. The IDI model is beneficial for developing students' mathematical skills and processes.	4.80	0.45	Very High
18. The IDI model is beneficial for fostering positive student attitudes toward learning mathematics.	4.60	0.55	Very High
19. The IDI model is beneficial for other mathematics teachers.	4.60	0.55	Very High
The IDI model is beneficial for teachers of other subjects.	4.60	0.55	Very High
Overall mean score	**4.64**	**0.53**	**Very High**
Overall mean across all dimensions	**4.64**	**0.53**	**Very High**